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An Upper Limit for $Br(Z^0 \rightarrow ggg)$ from Two and Three Jet Correlations in 3–jet Z^0 Hadronic Decays

Preliminary

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Abstract

An upper limit for $Br(Z^0 \rightarrow 3g)$ is obtained from a correlation method, which distinguishes statistically between quark and gluon jets by using the difference in their charged particle multiplicity distributions. From the sample of mirror symmetric three jet events collected by the DELPHI experiment at LEP during 1991-1995, the 95% confidence level upper limit is deduced to be: $Br(Z^0 \rightarrow 3g) \leq 2.4 \times 10^{-2}$. From the sample of threefold symmetric three jet events the 95% confidence level upper limit is obtained to be: $Br(Z^0 \rightarrow 3g) \leq 3.9 \times 10^{-3}$.

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1 Introduction

The measurement of branching ratio for the decay of the Z^0–boson into three gluons is a good test for the Standard Model which predicts a very small branching ratio for the decay $Z^0 \to 3g$ from quark loops [1]:

$$Br^{SM}(Z \to 3g) \ \simeq 2.0 \times 10^{-6} \tag{1}$$

and for the Model of Compositeness of the Z–boson which would induce new couplings and decay modes and a predicted branching ratio [2]:

$$Br(Z \to 3g) \ \leq 2.0 \times 10^{-3}, \tag{2}$$

much larger than the standard model expectation.

In recent DELPHI paper [3] an upper limit for $Br(Z \to 3g)$ has been determined from a sample of threefold symmetric 3 jet events in which the angles between jets are in the range $120 \pm 20^\circ$ (referenced below as M–events). The analysis is based on the difference between the charged particle multiplicity distributions of quark and gluon jets. This difference is exploited by comparing the correlations present between the jet multiplicities in symmetric 3 jet events, in general consisting of two quark jets and one gluon jet, to those in uncorrelated fake events constructed by mixing jets from different real events. The upper limit for $Br(Z^0 \to 3g)$ is found to be equal to 1.6×10^{-2}. This method, generally referred to as the correlation method, has also been applied to the study of the ratio of the mean charged particle multiplicities in gluon and quark jets in symmetric 3 jet events [4].

In present letter the correlation method is applied to the more abundant sample of mirror symmetric 3 jet events in which the two smaller energy jets have approximately equal energy in the range 25 ± 5 GeV (referenced below as Y–events). The method is modified in such a way that only correlation between charged particle multiplicities of smaller energy jets is considered. The final sample of M–events is processed by using the procedure described in previous work[3]. The data used were collected by the DELPHI experiment at LEP in the years 1991 to 1995 at centre-of-mass energies around 91.2 GeV. They consist of about 3.5 million hadronic Z^0 decays.

2 The correlation method

The multiplicity correlation function is defined as :

$$C(n_2, n_3) = \frac{P(n_2, n_3)}{P_{uncor}(n_2, n_3)}, \tag{3}$$

where $P(n_2, n_3)$ is the probability of observing a 3 jet event in which the charged particle multiplicities of the smaller energy jets are equal to n_2 and n_3. Jets will always be numbered such that $n_2 \geq n_3$. The charged particle multiplicity of the biggest energy jet, n_1, is ignored. $P_{uncor}(n_2, n_3)$ is the corresponding probability for uncorrelated jets constructed using the mixed event technique: one mixed Y–event was obtained from three different real 3 jet events by selecting one jet at random from each event.

Assuming the multiplicities of the individual jets in a real event to be uncorrelated, the probability $P(n_2,n_3)$ can be expressed through the multiplicity distributions for gluon jets, $G(n)$, light (uds) quark jets, $Q(n)$, c–quark jets, $O(n)$, and b–quark jets, $B(n)$, respectively:

$$\begin{gathered} P(n_2,n_3) = \tfrac{1-\beta-\zeta}{2}\{(1-R_c-R_b)[G(n_2)Q(n_3)+G(n_3)Q(n_2)] + \\ +R_c[G(n_2)O(n_3)+G(n_3)O(n_2)] + R_b[G(n_2)B(n_3)+G(n_3)B(n_2)]\} + \\ +\zeta\{(1-R_b-R_c)Q(n_2)Q(n_3) + R_bB(n_2)B(n_3) + R_cO(n_2)O(n_3)\} + \beta G(n_2)G(n_3), \end{gathered} \tag{4}$$

where $\beta = N^{sym}_{ggg}/N^{sym}_{3jet}$ is the fraction of three–gluon events and $1-\beta$ the fraction of $Z \to q\bar{q}g$ events in the symmetric 3 jet event sample, ζ is the fraction of events with gluon jet carrying highest jet energy in the event, $R_c = \Gamma_{c\bar{c}}/\Gamma_{had}$ and $R_b = \Gamma_{b\bar{b}}/\Gamma_{had}$ are the $Z \to c\bar{c}$ and $Z \to b\bar{b}$ branching fractions.

By construction, jets in the mixed event sample are completely uncorrelated. Therefore:

$$P_{uncor}(n_2,n_3) = J(n_2)J(n_3), \tag{5}$$

where

$$\begin{gathered} J(n) = \tfrac{1-\beta-\zeta}{2}[G(n) + (1-R_c-R_b)Q(n) + R_cO(n) + R_bB(n)] + \\ +\zeta[(1-R_c-R_b)Q(n) + R_cO(n) + R_bB(n)] + \beta G(n). \end{gathered} \tag{6}$$

The experimental correlation function $C(n_2,n_3)$ is determined by dividing the number of measured events with given n_2 and n_3 by the normalized number of such events from the mixed event sample.

The multiplicity correlation function for M–events sample is defined as following:

$$C(n_1,n_2,n_3) = \frac{P(n_1,n_2,n_3)}{P_{uncor}(n_1,n_2,n_3)}, \tag{7}$$

where $P(n_1,n_2,n_3)$ is the probability of observing a M–event in which the charged particle multiplicities of the jets are equal to n_1, n_2 and n_3. Jets will always be numbered such that $n_1 \geq n_2 \geq n_3$. $P_{uncor}(n_1,n_2,n_3)$ is the corresponding probability for uncorrelated jets constructed using the mixed event technique.

Under assumption that the multiplicities of the individual jets in a real event are uncorrelated, the probability $P(n_1,n_2,n_3)$ can be expressed through the multiplicity distributions for jets:

$$\begin{gathered} P(n_1,n_2,n_3) = \beta G(n_1)G(n_2)G(n_3) + \tfrac{1-\beta}{3} \times \\ \times\{(1-R_c-R_b)[G(n_1)Q(n_2)Q(n_3) + Q(n_1)G(n_2)Q(n_3) + Q(n_1)Q(n_2)G(n_3)] + \\ +R_c[G(n_1)O(n_2)O(n_3) + O(n_1)G(n_2)O(n_3) + O(n_1)O(n_2)G(n_3)] + \\ +R_b[G(n_1)B(n_2)B(n_3) + B(n_1)G(n_2)B(n_3) + B(n_1)B(n_2)G(n_3)]\}. \end{gathered} \tag{8}$$

The probability of generating the mixed event with n_1, n_2 and n_3 is given by the following formula:

$$P_{uncor}(n_1,n_2,n_3) = J(n_1)J(n_2)J(n_3), \tag{9}$$

where

$$J(n) = \frac{1-\beta}{3}\{G(n) + 2[(1-R_c-R_b)Q(n) + R_cO(n) + R_bB(n)]\} + \beta G(n). \qquad (10)$$

The experimental correlation function $C(n_1,n_2,n_3)$ is determined by dividing the number of measured events with given n_1, n_2 and n_3 by the normalized number of such events from the mixed event sample.

In the analysis the particle multiplicity distributions of gluon and quark jets, $G(n)$, $Q(n)$, $O(n)$ and $B(n)$, are assumed to be described by Negative Binomial Distributions (NBD)[6]. To cross–check that the results are not unduly sensitive to this assumption, a Poissonian parameterization (PD) of the shapes of the multiplicity distributions was also tried.

The unknown parameters were determined from a fit of the parametrized correlation function $C(n_2,n_3)$ ($C(n_1,n_2,n_3)$) as defined by equations 3–6 (7–10) to the measured one. The NBD parameters of b–quark jets,$\langle n\rangle_b$ and k_b, were obtained from a separate fit of the charged particle multiplicity distribution of the smaller energy b–tagged[5] jets in Y– and M–events samples. The NBD parameters of gluon jets,$\langle n\rangle_g$ and k_g, were obtained from a fit of the charged particle multiplicity distribution of the smaller energy not tagged as b jets in b–tagged events with the second smaller energy jet tagged as b jet. The NBD parameters of light–quark jets,$\langle n\rangle_q$ and k_q, were obtained from a fit of the charged particle multiplicity distribution of the smaller energy jets in uds–tagged events assuming that distribution is a superposition of gluon and light–quark jets. The parameter corresponding to the difference in mean multiplicity between c-quark and light quark jets was fixed according to the published data[7, 8, 9, 10, 11]. The NBD width parameters of c–quark jets, k_c, is assumed to be equal to that of light–quark jets, k_q. Therefore the finally fitted parameter is the fraction of 3–gluon events, β.

3 Experiment and data selection

A detailed description of the DELPHI detector can be found elsewhere [12]. In this analysis only charged particles were used. Their momenta were measured in the 1.2 T solenoidal magnetic field by the following tracking detectors: the Micro Vertex Detector, the Inner Detector, the Time Projection Chamber (TPC, the principal tracking device of DELPHI), the Outer Detector and the Forward Chambers A and B.

A charged particle was required to satisfy the following criteria :

- momentum, p, greater than 0.2 GeV/c;
- error on $p < p$;
- polar angle, θ, with respect to the beam between 25° and 155°;
- measured track length in the TPC greater than 50 cm;
- impact parameter with respect to the nominal beam crossing point within 5 cm in the transverse xy plane and 10 cm along the beam direction (z-axis).

Hadronic events from Z^0 decays were then selected if

– there were at least 5 charged particles;

– the total energy of charged particles (assuming a pion mass) in each of the two hemispheres defined with respect to the beam direction exceeded 3 GeV;

– the total energy of all charged particles was greater than 15 GeV.

A total of 3.5×10^6 events satisfied these cuts. The contamination from events due to beam-gas scattering and to $\gamma\gamma$ interactions was estimated to be less than 0.1% and the background from $\tau^+\tau^-$ events to be less than 0.3% of the accepted events [14].

Samples of events with three jets were selected by applying the DURHAM jet–finder (also known as the $k_\perp$ algorithm), with jet resolution parameter y_{min}=0.015 or 0.035. The DURHAM jet–finder is well defined in perturbation theory, allowing calculations to incorporate leading terms to all orders [8], and is widely used in experimental work. The value y_{min}=0.035 has an advantage with respect to smaller values of y_{min} because it gives the symmetric 3 jet event sample which is less contaminated by the events without hard gluon emission artificially split into 3 jet by the jet–finder.

Each reconstructed jet was required to contain at least 2 charged particle, to have the jet axis lying in the region $|\cos\theta| \leq 0.7$, to have a visible energy larger than 2 GeV and to have the energy calculated from the angular relation in the range 25±5 GeV or greater than 30 GeV. To eliminate non-planar events, the sum of the angles between the three jets was required to exceed 357°. The total numbers obtained using the DURHAM algorithm are 82994 at $y_{min} = 0.015$ and 54371 at $y_{min} = 0.035$.

Threefold symmetric 3–jet events of M–type where selected by projecting the jets into the 3–jet event plane and requiring the angles between them to be in the range 100° to 140°. The total numbers of events are 12030 at $y_{min} = 0.015$ and 13702 at $y_{min} = 0.035$.

4 Results

The values of the NBD parameters for charged particle multiplicity distributions in jets were obtained from the fit of respective distributions for jets selected by using b–tagging technique.

The observed charged particle multiplicity distributions were fitted with the convolution of NBD with the acceptance matrix:

$$f(n) = \sum_{m=m_{min}}^{m_{max}} A_{nm} P_m^{NBD}, \tag{11}$$

where A_{nm} is the probability to observe n particles in the jet when the multiplicity in the produced jet is equal to m. It was calculated as a ratio of the number of jets reconstructed with multiplicity n after DELPHI detector simulation program DELSIM[14] to the number of jets generated by JETSET[15] with multiplicity m for each energy interval.

The NBD parameters of b–quark jets were obtained from the NBD fit of the charged particle multiplicity distribution of the smaller energy jets with Negative Logarithm of Positive Probability (NLPP) for jet greater than 4. The purity of the sample of b jets is equal to 92%.

The NBD parameters of gluon jets were obtained from the NBD fit of the charged particle multiplicity distribution of the smaller energy jets with NLPP less than 1 when the second smaller energy jet is tagged as b jet. To cross–check the results of the fit the sample of events with both b jets tagged was also used.

The NBD parameters of light–quark jets were obtained from the fit of the charged particle multiplicity distribution of the smaller energy jets in uds–tagged events assuming that distribution is a superposition of gluon and light–quark jets. The sample of the uds events was obtained requiring the Maximum NLPP (MNLPP) in the event to be less than 1. The sample obtained with this cut consist of 83% uds, 14% c and 3% b events. The dependence of the purity of the sample on MNLPP is shown in fig.1.

The resulting values of the parameters of multiplicity distributions for b, uds and gluon jets are presented in table 1.

It is worthwhile to note that the variance of multiplicity distribution, D, related with $\langle n\rangle$ and k through the formula:

$$\frac{D^2}{\langle n\rangle^2} = \frac{1}{\langle n\rangle} + \frac{1}{k}, \tag{12}$$

for gluon jets is greater than for quark jets in the M–events sample, as it is expected from the oscillations of cumulant moments of parton multiplicity distributions inside a jet[13].

The average value of the difference between the mean charged particle multiplicity in c–quark jets and that in light quark jets, δ_{cl}, was calculated to be equal to 0.44±0.21, the weighted average of the measurement by OPAL[8] and SLD[11].

The ζ parameter was determined from HERWIG[17] generated events and found to be equal 0.05.

Table 1: Number of events, average energy of jet and NBD parameters of charged particle multiplicity distribution in jets for jet energy intervals $20 \leq E_{jet} \leq 30GeV$ and $25 \leq E_{jet} \leq 35GeV$ and for different y_{min} .

Jet	N_{ev}	E_{jet}, GeV	$\langle n\rangle \pm \sigma_{stat} \pm \sigma_{syst}$	$k \pm \sigma_{stat} \pm \sigma_{syst}$	$P(\chi^2)$
		DURHAM, $y_{min} = 0.015$			
b	9091	25.8 ± 2.8	8.80±0.04 ± 0.05	$80\pm^{14}_{10}\pm^{10}_{5}$	0.20
g	2172	24.0 ± 2.8	9.32±0.08 ± 0.05	$59\pm^{16}_{10}\pm^{15}_{10}$	0.51
uds	24851	25.0 ± 2.9	6.83±0.05 ± 0.05	$19.5\pm^{2.1}_{1.7}\pm^{2.0}_{1.5}$	0.07
b	7755	30.5 ± 2.8	9.25±0.04 ± 0.08	$74\pm^{17}_{12}\pm^{11}_{8}$	0.28
g	2470	29.0 ± 2.8	10.22±0.08 ± 0.05	$30\pm^{4}_{3}\pm^{2}_{1.5}$	0.69
uds	13082	30.0 ± 2.9	6.25±0.06 ± 0.05	$72\pm^{51}_{21}\pm^{30}_{10}$	0.67
		DURHAM, $y_{min} = 0.035$			
b	5938	25.9 ± 2.8	9.22±0.05 ± 0.05	$75\pm^{25}_{15}\pm^{10}_{6}$	0.42
g	1545	24.4 ± 2.8	9.92±0.07 ± 0.05	$97\pm^{83}_{31}\pm^{21}_{10}$	0.15
uds	16216	25.1 ± 2.9	7.02±0.07 ± 0.05	$20.0\pm^{4.5}_{3.2}\pm^{2.5}_{1.5}$	0.07
b	5216	30.4 ± 2.8	9.63±0.05 ± 0.04	$85\pm^{30}_{18}\pm^{10}_{5}$	0.77
g	2039	29.1 ± 2.7	10.79±0.09 ± 0.06	$28\pm^{4.5}_{3.4}\pm^{4}_{2}$	0.93
uds	8538	29.9 ± 2.8	6.66±0.07 ± 0.05	$40\pm^{16}_{9}\pm^{11}_{7}$	0.17

In order to correct for the influence of imperfections of the DELPHI detector, the correlation method was applied to the samples of simulated events from the DELPHI detector simulation program DELSIM [14]. In DELSIM, events were generated using the JETSET 7.3 PS program [15] with DELPHI default parameters [16]. Particles were followed through the detector and the resulting simulated digitizations were processed with the same reconstruction programs as the experimental data.

Detector imperfections introduce a systematic difference between $C_J(n_2, n_3)$ for the events generated by JETSET and $C_D(n_2, n_3)$ for the events reconstructed after DELSIM (i.e. after the detector simulation). In order to correct for this influence of the detector, the correlation function $C(n_2, n_3)$ observed for uncorrected data was multiplied by the ratio $K(n_2, n_3) = C_J(n_2, n_3)/C_D(n_2, n_3)$.

In order to take into account the imperfections of the jet finder algorithms, a further correction factor was introduced. It was calculated as a ratio $N(n_2, n_3) = C_{expected}(n_2, n_3)/C_{observed}(n_2, n_3)$ for a normalisation sample of events obtained by generating symmetric $Z^0 \to q\bar{q}g$ decays using JETSET. This correction is based on the fundamental property that the correlation function should equal unity, i.e. $C_{expected}(n_2, n_3) = 1$, when the mixed events are constructed from the same numbers of quarks and gluons as real events. Indeed the probabilities $P(n_2, n_3)$ and $P_{uncor}(n_2, n_3)$ both are described by the formula (4) in this case. The total correction factor $K \cdot N$ is typically between 0.9 and 1.1.

The corrected correlation function $C(n_2, n_3)$ is presented as a function of n_3 in Fig.2 for the DURHAM jet–finder with $y_{min} = 0.015$ for several n_2 values. The curves in Fig.2 are the results of the fit for all values of $2 \leq n_2 \leq 25$. The numerical results of the fit are as follows. The value of β is equal to -0.010±0.022 with probability of the fit equal to 0.14 for 272 experimental points for y_{min} equal to 0.015 and β is equal to -0.032±0.037 with probability of the fit equal to 0.041 for 251 experimental points for y_{min} equal to 0.035.

In order to estimate the systematic errors due to the uncertainties in the values of the fixed parameters, the fit was also performed for the central values of these parameters plus or minus one standard deviation. The corresponding systematic errors in β are detailed in Tables 2 and 3.

Table 2: Contributions to the systematic error in β from the uncertainties in the parameters fixed in the fits for Y–events sample at $y_{min} = 0.015$.

Parameter value ± error	σ_{syst} NBD	σ_{syst} PD
$\langle n_g \rangle / \langle n_q \rangle = 1.37 \pm 0.02$	$^{+0.048}_{-0.054}$	$^{+0.040}_{-0.044}$
$\delta_{bl} = 1.97 \mp 0.09$	$^{+0.013}_{-0.013}$	$^{+0.010}_{-0.009}$
$\delta_{cl} = 0.44 \mp 0.21$	$^{+0.018}_{-0.018}$	$^{+0.016}_{-0.016}$
$k_q = 19.5 \pm^{2.9}_{2.3}$	$^{+0.011}_{-0.010}$	
$k_g = 59 \pm^{22}_{14}$	$^{+0.013}_{-0.011}$	
$k_b = 80 \pm^{17}_{11}$	$^{+0.001}_{-0.001}$	
Total	$^{+0.056}_{-0.060}$	$^{+0.044}_{-0.048}$

Further systematic errors were estimated taking into account the variation of the results obtained with different cuts on the jet multiplicity n_2 and the uncertainty in the values of the total correction coefficients. The resulting systematic bias in the values of β does not exceed 0.004 and 0.002 for y_{min} equal to 0.015 and 0.035 respectively.

Including the systematic errors in β leads to the following final results for β:

$$\begin{aligned}\beta &= -0.010 \pm 0.022(stat.) \pm^{0.056}_{0.060} (syst.) \quad (y_{min} = 0.015)\\ \beta &= -0.032 \pm 0.037(stat.) \pm^{0.054}_{0.060} (syst.) \quad (y_{min} = 0.035).\end{aligned}$$

The branching fraction $Br(Z^0 \to ggg)$ is calculated from β using the following formula:

$$Br(Z^0 \to 3g) = \beta \cdot Br(Z^0 \to hadr) \cdot \frac{N_{3jet}^{sym}}{N_{hadr}} \cdot \frac{N_\Upsilon}{N_\Upsilon^{sym}}, \tag{13}$$

where N_{3jet}^{sym}/N_{hadr} is the fraction of symmetric 3 jet events in the hadronic event sample and $N_\Upsilon^{sym}/N_\Upsilon$ is the fraction of symmetric decays in an Υ–like 1^{--} quarkonium state to three gluons. The latter ratio was calculated using JETSET 7.3. The mass of the pseudo-onium was chosen to be equal to the Z mass. Due to the identical helicity structure of $Z^0 \to ggg$ and $\Upsilon \to ggg$ decays, the angular distributions for jets from the two sources are expected to be identical. Thus N_{ggg}^{sym}/N_{ggg} should equal $N_\Upsilon^{sym}/N_\Upsilon$. The numerical value of the factor relating $Br(Z \to 3g)$ to β in eq.(13) was found to be 0.263 at y_{min}=0.015 and 0.184 at y_{min}=0.035.

To calculate the 95% confidence level upper limits on the branching fraction $Br(Z^0 \to ggg)$, the systematic errors were added in quadrature to the statistical errors and unphysical negative values of β were forced up to have $\beta = 0$. The calculation gave:

$$UL\{Br(Z \to 3g)\} = 0.032 \quad (y_{min} = 0.015)$$

and

$$UL\{Br(Z \to 3g)\} = 0.024 \quad (y_{min} = 0.035).$$

For the limit calculation, the systematic and statistical errors on β were assumed to be uncorrelated.

The cross-check of using the Poissonian parametrisation of the multiplicity distributions gave similar estimates of the upper limit, namely 0.053 and 0.035 respectively with the probability of the fit at the level of 3×10^{-3} at $y_{min} = 0.015$ and 0.07 at $y_{min} = 0.035$.

The analysis of the increased statistics of the M–events of the DELPHI experiment was performed. In contrast with previous work[3] the difference in average charged particle multiplicities in c–quark jet and light quark jet is taken into account according to eqn.7-10.

The contributions to the systematic error in β from the uncertainties in the parameters fixed in the fits are collected in Tables 4 and 5.

The fit and the systematic errors in β give the following final results for β:

$$\begin{aligned}\beta &= -0.009 \pm 0.014(stat.) \pm^{0.016}_{0.017} (syst.) \quad (y_{min} = 0.015)\\ \beta &= -0.003 \pm 0.013(stat.) \pm^{0.015}_{0.019} (syst.) \quad (y_{min} = 0.035).\end{aligned}$$

The probability of the fit is equal to 0.97 for 235 experimental points for y_{min} equal to 0.015 and 0.98 for 242 experimental points for y_{min} equal to 0.035. The numerical value of the factor relating $Br(Z \to 3g)$ to β in eq.(13) was found to be 0.144 at y_{min}=0.015 and 0.100 at y_{min}=0.035.

The 95% confidence level upper limit for the $Z^0 \to ggg$ branching ratio is obtained as follows:

$$UL\{Br(Z^0 \to 3g)\} = 0.0060 \quad (y_{min} = 0.015)$$

and

$$UL\{Br(Z^0 \to 3g)\} = 0.0039 \quad (y_{min} = 0.035).$$

The Poissonian parametrisation of the multiplicity distributions gave similar estimates of the upper limit, namely 0.0052 and 0.0040 respectively with acceptable probability of the fit in both cases.

As a final result of the present study, the value of the upper limit from the M–events sample at $y_{min} = 0.035$ is accepted because in this sample the jetfinder algorithm gives the best separation of the jets. The upper limit is set four times lower the published paper[3] due to increased statistics of the experiment and reduced number of free parameters of the fit.

5 Summary

By using a correlation method based on the difference between the particle multiplicity distributions of quark and gluon jets, an upper limit at 95% confidence level for the $Z^0 \to ggg$ branching ratio has been established from the sample of Y–events:

$$Br(Z^0 \to 3g) \leq 3.2 \times 10^{-2}$$

for the DURHAM jet–finder with y_{min}=0.015 and

$$Br(Z^0 \to 3g) \leq 2.4 \times 10^{-2}$$

with y_{min}=0.035.

The correlation method modified for the case of two jet correlations described in present paper can be applied in principle to esimate the possible contribution in $2jet + \gamma$ event sample from the process $Z^0 \to \gamma gg$ which has the one order of magnitude larger cross section than that for $Z^0 \to 3g$ in the compositeness model.

The correlations between multiplicities of 3 jets give more restrictive values for the upper limit:

$$Br(Z \to 3g) \leq 6.0 \times 10^{-3}$$

for the DURHAM jet–finder with y_{min}=0.015 and

$$Br(Z \to 3g) \leq 3.9 \times 10^{-3}$$

with y_{min}=0.035.

At the present level of statistics, no signal of the decay $Z^0 \to ggg$ expected from the compositeness model is observed.

The variance of multiplicity distribution for gluon jets is greater than for quark jets in the M–events sample, as it is expected from the oscillations of cumulant moments of parton multiplicity distributions inside a jet.

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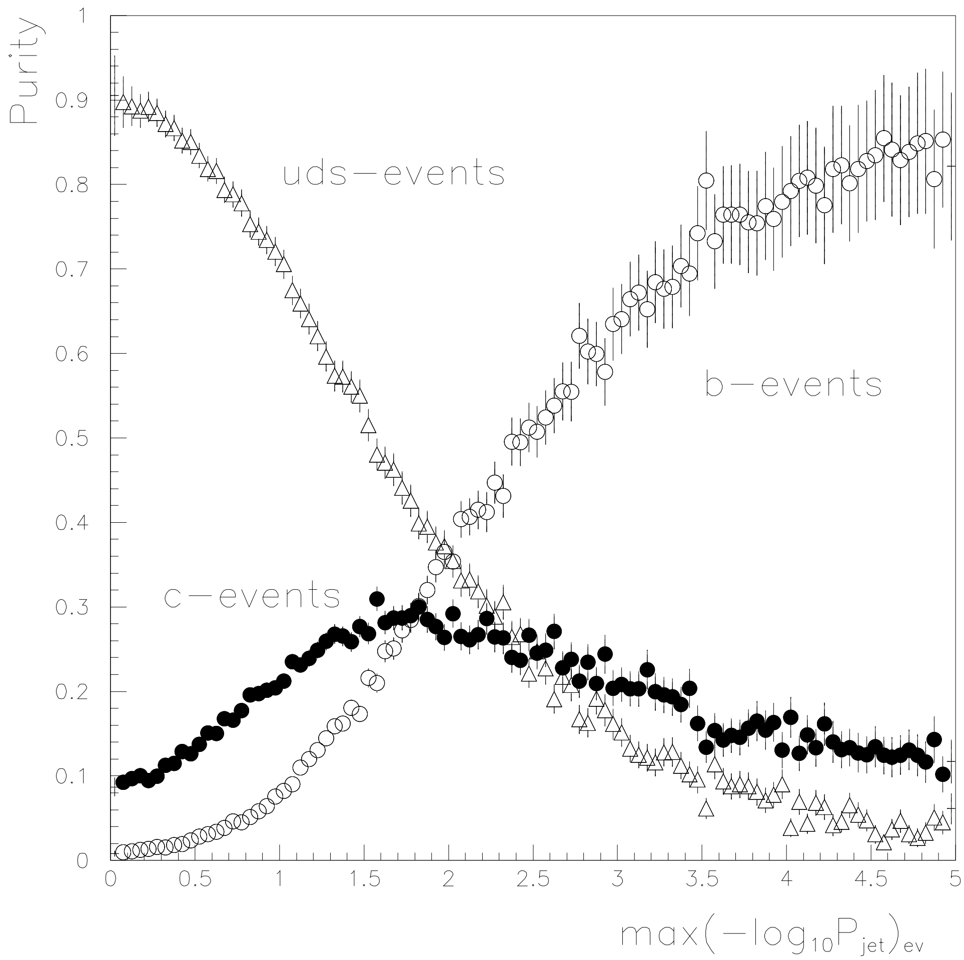

Figure 1: The purity of the sample of events as a function of the maximum of negative logarithm of positive probability for jets in the event.

Table 3: Contributions to the systematic error in β from the uncertainties in the parameters fixed in the fits for Y–events sample at $y_{min} = 0.035$.

Parameter value ± error	σ_{syst} NBD	σ_{syst} PD
$\langle n_g\rangle/\langle n_q\rangle = 1.41 \pm 0.02$	$^{+0.045}_{-0.052}$	$^{+0.047}_{-0.041}$
$\delta_{bl} = 2.20 \mp 0.11$	$^{+0.013}_{-0.013}$	$^{+0.011}_{-0.011}$
$\delta_{cl} = 0.44 \mp 0.21$	$^{+0.015}_{-0.016}$	$^{+0.013}_{-0.013}$
$k_q = 20.0\pm^{5.1}_{3.5}$	$^{+0.015}_{-0.014}$	
$k_g = 97\pm^{86}_{33}$	$^{+0.018}_{-0.016}$	
$k_b = 75\pm^{27}_{16}$	$^{+0.001}_{-0.001}$	
Total	$^{+0.054}_{-0.060}$	$^{+0.050}_{-0.044}$

Table 4: Contributions to the systematic error in β from the uncertainties in the parameters fixed in the fits for M–events sample at $y_{min} = 0.015$.

Parameter value $\pm$ error	σ_{syst} NBD	σ_{syst} PD
$\langle n_g\rangle/\langle n_q\rangle = 1.64 \pm 0.02$	$^{+0.010}_{-0.011}$	$^{+0.003}_{-0.004}$
$\delta_{bl} = 3.00 \mp 0.12$	$^{+0.012}_{-0.012}$	$^{+0.004}_{-0.004}$
$\delta_{cl} = 0.44 \mp 0.21$	$^{+0.002}_{-0.002}$	$^{+0.002}_{-0.002}$
$k_q = 72\pm^{60}_{23}$	$^{+0.001}_{-0.001}$	
$k_g = 30.0\pm^{4.5}_{3.4}$	$^{+0.004}_{-0.003}$	
$k_b = 74\pm^{20}_{14}$	$^{+0.001}_{-0.001}$	
Total	$^{+0.016}_{-0.017}$	$^{+0.005}_{-0.005}$

Table 5: Contributions to the systematic error in β from the uncertainties in the parameters fixed in the fits for M–events sample at $y_{min} = 0.035$.

Parameter value $\pm$ error	σ_{syst} NBD	σ_{syst} PD
$\langle n_g\rangle/\langle n_q\rangle = 1.62 \pm 0.03$	$^{+0.011}_{-0.014}$	$^{+0.011}_{-0.013}$
$\delta_{bl} = 2.97 \mp 0.11$	$^{+0.009}_{-0.011}$	$^{+0.008}_{-0.009}$
$\delta_{cl} = 0.44 \mp 0.21$	$^{+0.001}_{-0.003}$	$^{+0.002}_{-0.003}$
$k_q = 40\pm^{19}_{11}$	$^{+0.001}_{-0.002}$	
$k_g = 28\pm^{6}_{4}$	$^{+0.003}_{-0.004}$	
$k_b = 85\pm^{32}_{19}$	$^{+0.001}_{-0.001}$	
Total	$^{+0.015}_{-0.019}$	$^{+0.014}_{-0.016}$

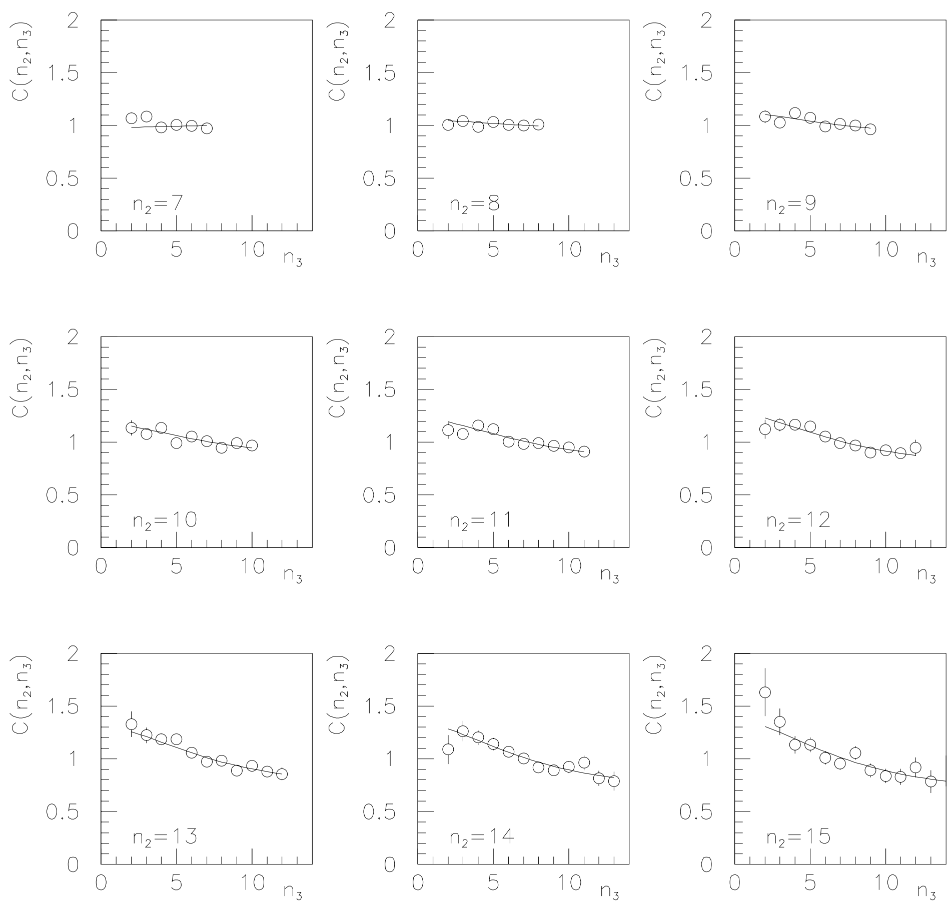

Figure 2: The corrected correlation function $C(n_2, n_3)$ as a function of the smallest jet multiplicity n_3 for different values of the jet multiplicity n_2. Symmetric 3–jet events are selected from the sample of DELPHI data by using the DURHAM jet–finder with y_{min} equal to 0.015. The curves are the result of the fit.